

# Frustration in Lattice Gauge Theory

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We introduce a  $U(1)$  lattice gauge theory which incorporates explicit frustration in  $d > 2$ . We show, by identifying an appropriate order parameter and through computer simulations, the existence of a frustrated region in the phase diagram of the model. We study this phase diagram and the nature of the transition lines.

## 1. Introduction

The importance of frustration and disorder is well known to condensed-matter physicists working in the field of spin-glasses and related systems [1]. In these systems one can find a variety of “unusual” phases which, in some cases, present completely new phenomena (for example, the existence of a multiplicity of vacua not related by a symmetry of the action). It is interesting to ask whether similar phenomena could happen in a field theory and what will be the physical consequences.

As a first step in this direction, we address here the issue of frustration in lattice gauge theories. Specifically we will study a model in which frustration is introduced by hand (although inspiration is taken from the hopping-parameter expansion of QED).

## 2. Definition of the model

The model we have studied presents explicit frustration. It is defined by the following action:

$$S = S_4 + S_6 = -\beta \sum_{pl} \text{Re} U_{pl} + \beta_6 \sum_{p_6} \text{Re} U_{p_6} \quad (1)$$

with  $\beta, \beta_6 \geq 0$ . The first part is the standard  $U(1)$  pure gauge Wilson action, whereas the second part is defined as a sum of contributions over all closed loops made up with six (non-repeated) links. These loops can be classified in three different classes: planar loops, loops that involve two

planes and loops that involve three planes. In the special case  $d = 2$  only the planar loops appear, which implies the absence of frustration for this system, consistently with the fact that it can be mapped onto the  $X - Y$  model within an external field.

## 3. Classical ground states and order parameter

It is not difficult to realize the existence of frustration in this model at the classical level and in  $d \geq 3$ , even if we consider only the  $S_6$  piece. In fact, it is not possible to minimize simultaneously the contributions to action (1) of all three classes of loops: the best we can do is to minimize the planar and three-planes loop contributions using some special chess-board-like configurations, that we will call “antiferromagnetic”, with plaquettes taking values alternatively  $\pm 1$ . The two-planes loop contribution is however not minimized in this way.

We have studied in more detail the case  $d = 4$ . All the results reported here correspond to this case. We have checked the relevance of the states described above by doing simulations at large values of  $\beta_6$ . The numerical results show that the system tends to freeze in one of these configurations. This suggests the introduction of a new order parameter for each plane, the staggered plaquette, defined as follows:

$$P_{\mu\nu}^s = \frac{1}{V} \sum_x \epsilon(x) \text{Re} U_{\mu\nu}(x) \quad (2)$$

$$\epsilon(x) = (-1)^{x_1+x_2+x_3+x_4}$$

\*Talk presented by E. Follana.

This order parameter is different from zero in the antiferromagnetic vacuum, and vanishes in the ferromagnetic one.

#### 4. Numerical simulations

We have performed Montecarlo simulations of this model, using a standard Metropolis algorithm. The observables we measure in the simulation are, in addition to the staggered plaquette, the following two quantities:

$$P = \frac{1}{6V} \sum_{pl} \text{Re} U_{pl} \quad (3)$$

$$P_6 = \frac{1}{76V} \sum_{p_6} \text{Re} U_{p_6} \quad (4)$$

The first one is the usual normalized plaquette, and the second one is the normalized contribution of the 6-loop part of the action.

##### 4.1. The line $\beta = 0$

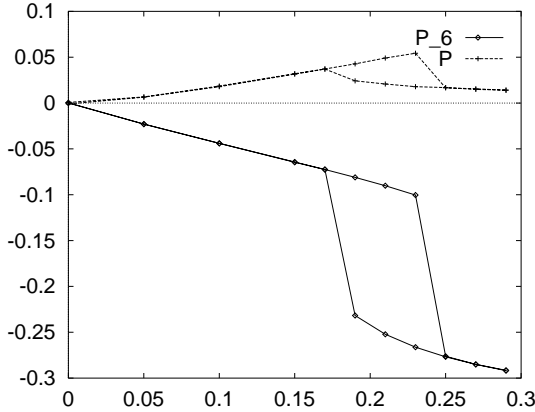


Figure 1. Plaquette and 6-loop hysteresis cycles at  $\beta = 0$  against  $\beta_6$

We show in figures 1 and 2 the annealing cycles at  $\beta = 0$ . We see a clear hysteresis effect signaling a strong first-order phase transition. This hysteresis cycle does not show any significant change when we increase the simulation time over two

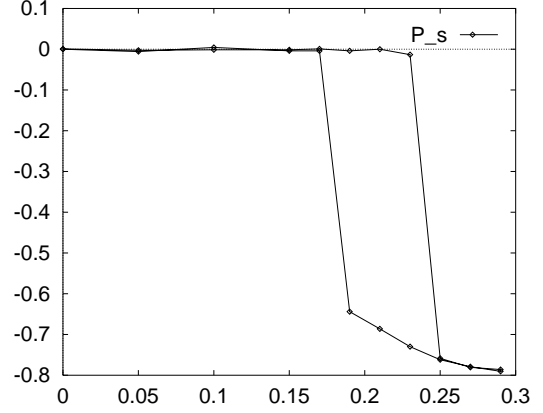


Figure 2. Staggered Plaquette hysteresis cycle at  $\beta = 0$  against  $\beta_6$

orders of magnitude, neither when we combine the Metropolis algorithm with an over-relaxation procedure.

We can see in figure 2 that the staggered plaquette is in fact an appropriate order parameter for the transition to the antiferromagnetic phase.

##### 4.2. The lines of constant $\beta$

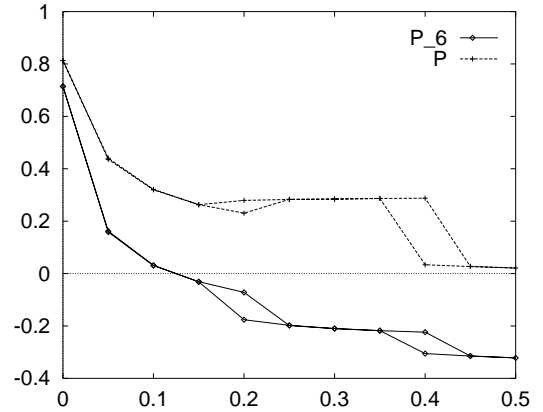


Figure 3. Plaquette and 6-loop hysteresis cycles at  $\beta = 1.5$  against  $\beta_6$

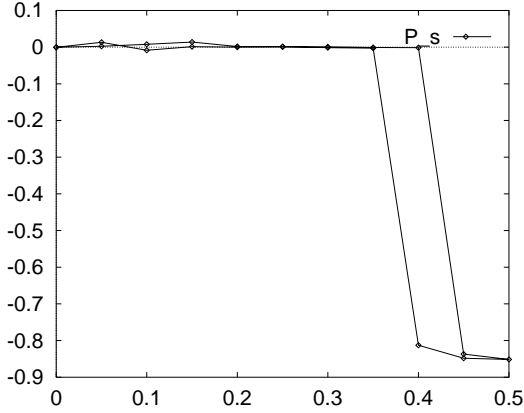


Figure 4. Staggered Plaquette hysteresis cycle at  $\beta = 1.5$  against  $\beta_6$

In figures 3 and 4 we report the results obtained by cycling over  $\beta_6$  while keeping a fixed  $\beta = 1.5$ , which is well above the value for the usual confined-deconfined phase transition for the standard  $U(1)$  pure gauge theory. We can clearly see here two transitions: the first one corresponds to the continuation of the usual confined-deconfined transition (the staggered plaquette remains zero for this transition), whereas the second one corresponds to the transition to the antiferromagnetic phase, as clearly shown by the jump of the staggered plaquette. This results are also stable against changes in the Montecarlo time.

#### 4.3. Phase diagram

We show in figure 5 the tentative phase diagram (restricted to the positive quadrant in the  $\beta - \beta_6$  plane) extracted from our simulations of the model described by action (1). We found three phases separated by two first-order lines. The antiferromagnetic phase is characterized by a non-zero value of the staggered plaquette, which is zero in the other two phases. In this phase we have several states, to be precise eight states, related by (spontaneously broken) symmetries of the action. The other two phases are the continuation of the confined and unconfined phases of the standard compact pure gauge model.

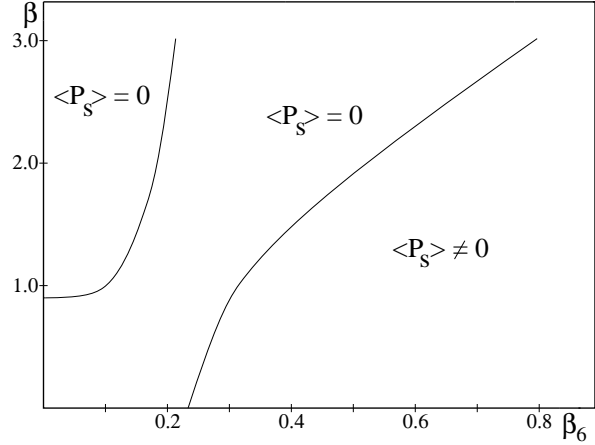


Figure 5. Phase diagram

#### 5. Comments

We have analyzed here the simplest gauge invariant abelian frustrated model and shown how frustration plays a fundamental role in the dynamics and the vacuum structure. This work is a first step of the more ambitious program of investigating possible implications of frustration in gauge theories with dynamical fermions. There are several not very well understood phenomena in QED, as the strong coupling phase transition in 3+1 dimensions [2,3] and the “apparent” transition in 2+1 dimensions [4], the origin of which could be related to the frustrated character of the effective fermionic action.

#### REFERENCES

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